**Non-Equilibrium Properties**

Now we’ll take a look at some non-equilibrium properties of a superconductor. We’ll use the mean field Hamiltonian,



where,



and where A is the vector potential, and jp the paramagnetic current density,



**Conductivity Tensor**

And we’ll sketch out a calculation of the conductivity and closely related absorption. So in the Metals/Impurities/Nonequilibrium/Conduction file, we derived an expression for the conductivity tensor:



where the Π guy is the retarded current-current correlation function:



(Heq would be our H above, sans A terms) Normally we’d just calculate the complex time ordered guy and then analytically continue.

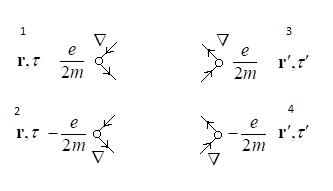


And from the definition of the current density (well, paramagnetic current density), we have:

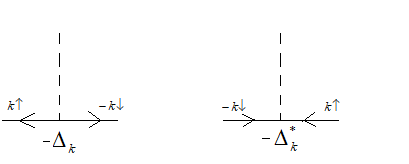


**Diagrammatic Analysis**

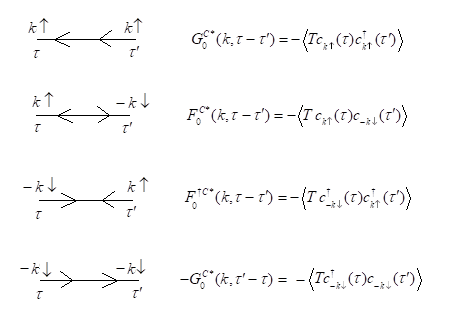
Like we did in that file referenced above, we can represent the four pairs of ψ’s with the following external vertices:



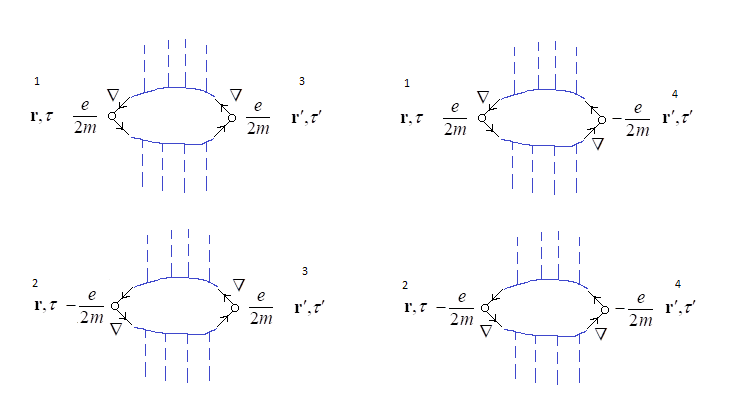
We needn’t worry about contractions between vertices with the same time as this will basically give us two copies of the equilibrium current, which is zero. So we only need to focus on contractions between brackets. The contractions would involve the following vertices from the interaction part of the Hamiltonian,



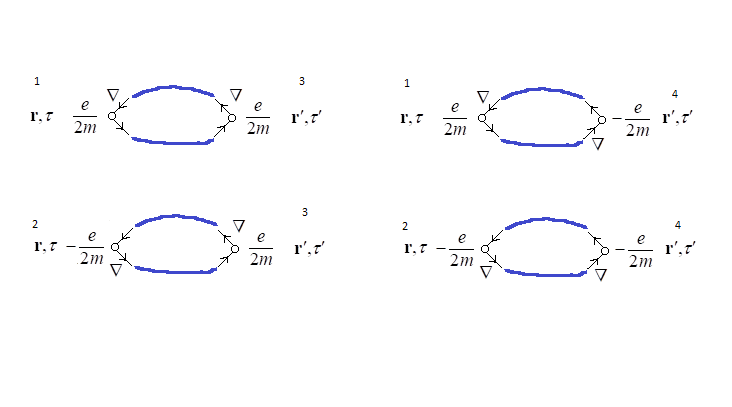
and then we’d have the following GF’s (I don’t actually know what these ‘unperturbed’ GF’s would be as we only calculated the perturbed ones – I guess they’d be the Mean Field G, F, F†, and the other G, with Δ set to 0)



Note that we can connect to either end of two potential/interaction vertices, since we have <cc> and <c†c†> GF’s now. I think we can interpret the labels on the vertices as just stipulating that we should flip the sign of the incoming momentum to its opposite when outgoing. Okay let’s consider some of the diagrams we can get. So first, we’ll take a look at the diagrams that are connected top-top and bottom-bottom:



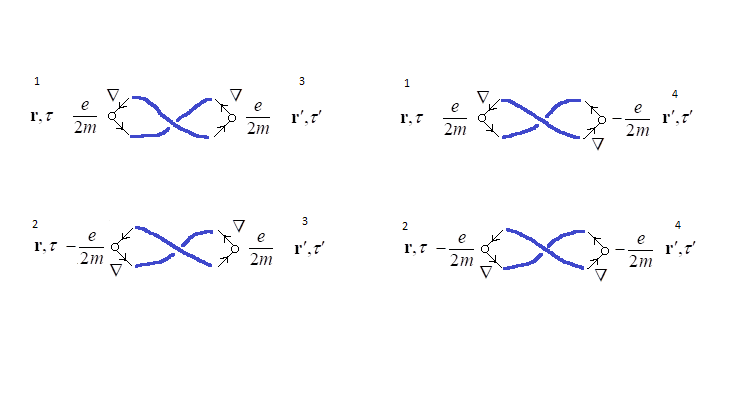
I drew 4 interaction vertices on each line, but there could be any number of them. And, eliding a few details, the sum of all diagrams with any number of interaction lines basically gives us the fully developed GF on each of those lines. So, we have:



Following the arguments in that Metals/Impurities/Nonequilibrium/Conduction/Quantum file aforementioned, the sum of these diagrams will work out to:



We can connect the vertex ends in reverse, adding a bunch of interaction lines, etc., and this will result in the following set of diagrams.



which comes to, hopefully, because I don’t want to think about it:



and so altogether we would have:



I guess the spin subscripts don’t make any difference, so I’ll take them off…and then we have:



This is a little fishy. Let’s do this another way.

**Non-diagrammatic analysis (don’t know what else to call it)**

We could straight away try to evaluate the product,



now this should be a function of r1 – r2, which would require the correlation function to be proportional to δq1,-q2, which gives us:



Then taking the Fourier transform,



So now filling in jαp(q) (see QM/ManyParticles/2nd Quant), we have:



We can use Wick’s theorem because the Hamiltonian is bilinear in creation/annihilation operators. So,



I think I’ll explicitly fill in the spin sum, and cross out the guys which can’t exist (because they don’t conserve spin)



And now, since we have a translationally invariant system, the expectations should also conserve momentum, so doing that:



Now gonna put the correlations into GF form,



which, recalling,



is,



But the GF’s are even in momentum argument, and going to flip sign of q, since Π shouldn’t care, i.e., is real in position space. So,



Guess I’ll take temporal FT now,



So we can say,



We can put this in the form we got from our ad hoc diagrammatic analysis. Consider the third F term: we’ll change variables of of summation ωm → ωm – νn,



and then since Π should be a real function of t, we should get the same thing if we say iνn → -iνn.

Then we’e have:



So this term is then equal to the second one, and we would now have:



which matches the diagrammatic expansion expression.

**In terms of Nambu matrix GF**

We can also, conveniently, write this in terms of the Nambu GF that we defined in the GF MFT file. So go back to:



and recall the Nambu GF was, and still is:



We’ll allege,



Expanding it,



This is almost what we need. Consider the last two terms, and change variables by subtracting q from k (this won’t change the range of summation)



where in the last line we changed variables again q → -q (should be able to do this without affecting ΠC\* as it should be real in position space) . So now the whole thing comes to:



which is what we got before in the non-diagrammatic analysis section. So cool. Now let’s take the Fourier transform of our Nambu expression,



which brings us to,



and we’ll recall from GF file,



Now ultimately, we’re interested in the absorbtivity and the DC conductivity. The absorbtivy is just the Real part of σ(q,ω) (assuming no dielectric response – see EM folder/Maxwell’s Equations in Metal-Insulator. So we want,

